I. Gaussovou eliminační metodou řešte soustavu lineárních rovnic:

1. $$\begin{aligned} 3x_1 + x_2 - 2x_3 &= 4 \\ x_1 \qquad + x_3 &= 2 \\ 2x_1 - x_2 + 3x_3 &= -2 \end{aligned}$$ $(0,8,2)$

2. $$\begin{aligned} x + 2y + 3z + 4u &= -2 \\ 2x + 3y + 4z + u &= 2 \\ 3x + 4y + z + 2u &= -2 \\ 4x + y + 2z + 3u &= 2 \end{aligned}$$ $(1,-1,1,-1)$.

3. $$\begin{aligned} x + 2y + z - u &= 1 \\ 2x + 3y - z + 2u &= 3 \\ 4x + 7y + z &= 5 \\ 5x + 7y - 4z + 7u &= 8 \end{aligned}$$ $(3+5p-7q, -1-3p+4q, p, q)$, $p \in \mathbf{R}$, $q \in \mathbf{R}$

4. $$\begin{aligned} x_1 + 2x_2 + 3x_3 - x_4 &= 0 \\ x_1 + 5x_2 + 5x_3 - 4x_4 &= -4 \\ x_1 - x_2 + x_3 + 2x_4 &= 4 \\ x_1 + 8x_2 + 7x_3 - 7x_4 &= 6 \end{aligned}$$ soustava nemá řešení

5. $$\begin{aligned} 2x - 3y + 4z - w &= 1 \\ 2x - 3y + 2z + 3w &= 2 \\ 2x - 3y + 2z - 11w &= -4 \end{aligned}$$ např. $\left(p, \frac{2}{3}p + \frac{4}{3}, \frac{3}{2}, 1\right)$, $p \in \mathbf{R}$

6. $$\begin{aligned} 2x_1 - 2x_2 + 4x_3 + 2x_4 &= 3 \\ x_1 - x_2 + 3x_3 + x_4 &= 2 \\ x_1 + x_2 - x_3 + x_4 &= 1 \\ 2x_1 + 3x_2 + x_3 + 2x_4 &= 4 \end{aligned}$$ např. $\left(1 - x_4, \frac{1}{2}, \frac{1}{2}, x_4\right)$, $x_4 \in \mathbf{R}$

7. $$\begin{aligned} 7x - 9y + 2z + 4w &= 5 \\ 3x - 6y + z - 5w &= 2 \\ 8x - 11y - 2z + w &= 6 \\ 2x - 4y + 5z - 2w &= 9 \end{aligned}$$ soustava nemá řešení

8. $$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 2 \\ 2x_1 + 5x_2 - 2x_3 + 2x_4 &= 6 \\ 4x_1 + 9x_2 - 4x_3 + 2x_4 &= 6 \\ 5x_1 + 12x_2 - 5x_3 + 3x_4 &= 10 \end{aligned}$$ $(x_1, 2, 4 + x_1, 2)$, $x_1 \in \mathbf{R}$ nebo $(x_3 - 4, 2, x_3, 2)$, $x_3 \in \mathbf{R}$

9. $$\begin{aligned} 2x_1 + x_2 - x_3 + x_4 + x_5 &= 1 \\ x_1 - x_2 + x_3 + x_4 - 2x_5 &= 0 \\ 3x_1 + 3x_2 - 3x_3 + 3x_4 + 4x_5 &= 2 \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 &= 3 \end{aligned}$$ např. $\left(\frac{1}{3} + \frac{1}{3}x_5, \frac{1}{3} + x_3 - \frac{5}{3}x_5, x_3, 0, x_5\right)$, $x_3, x_5 \in \mathbf{R}$

$$
\begin{aligned}
&10. \quad
\begin{aligned}
x + y + 2z - u &= 6\\
x + 2y - z + u &= -5\\
2x - y + z + u &= 3\\
-x + y + z + 2u &= -4\\
x + 2z + 3u &= -1
\end{aligned}
\qquad (1, -1, 2, -2)
\end{aligned}
$$

II. Soustavu rovnic řešte úplnou eliminační metodou (Jordanova):

1.
$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 &= 2\\
2x_1 + 3x_2 + 2x_3 + 4x_4 &= 8\\
2x_1 + 2x_2 + x_3 - 3x_4 &= 1\\
x_1 + x_2 + x_3 - x_4 &= 0
\end{aligned}
\qquad (1, 2, -2, 1)
$$

2.
$$
\begin{aligned}
3x_1 + x_2 - x_3 + 2x_4 &= 0\\
x_1 + 2x_2 + x_3 - x_4 &= 0\\
2x_1 - x_2 + 2x_3 + x_4 &= 0\\
x_1 + 3x_2 + x_3 + 3x_4 &= 0
\end{aligned}
\qquad (0, 0, 0, 0)
$$

3.
$$
\begin{aligned}
x_1 - x_2 + x_3 &= 2\\
2x_1 - 3x_2 + 4x_3 &= 4\\
x_1 - x_3 &= 2
\end{aligned}
\qquad (t + 2, 2t, t),\ t \in \mathbf{R}
$$

4.
$$
\begin{aligned}
x_1 + x_2 - 3x_4 - x_5 &= 0\\
x_1 - x_2 + 2x_3 - x_4 &= 0\\
4x_1 - 2x_2 + 6x_3 + 3x_4 - 4x_5 &= 0\\
2x_1 + 4x_2 - 2x_3 + 4x_4 - 7x_5 &= 0
\end{aligned}
\qquad \left(\frac{7}{2}x_4 - x_3,\ x_3 + \frac{5}{2}x_4,\ x_3,\ x_4,\ 3x_4\right),\quad x_3, x_4 \in \mathbf{R}
$$

5.
$$
\begin{aligned}
2x_1 - 3x_2 - 2x_3 + x_4 &= 3\\
x_1 - x_2 - x_3 - x_4 &= 2\\
x_1 - 2x_2 - x_3 + 2x_4 &= 1\\
2x_1 + 2x_3 + x_4 &= 1
\end{aligned}
\qquad \left(\frac{7}{4} - \frac{7}{4}t,\ 1 + 3t,\ -\frac{5}{4} - \frac{9}{4}t,\ t\right),\ t \in \mathbf{R}
$$

III. Pomocí Cramerových vzorců řešte soustavu rovnic, která má právě jedno řešení:

1.
$$
\begin{aligned}
x + y + 3z &= 7\\
x - 3y + 2z &= 5\\
x + y + z &= 3
\end{aligned}
\qquad (1, 0, 2)
$$

2.
$$
\begin{aligned}
3x_1 + x_2 - 2x_3 &= 4\\
x_1 + x_3 &= 2\\
2x_1 - x_2 - x_3 &= -2
\end{aligned}
\qquad (4, 18, 2)
$$

3. $\begin{aligned} 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + 2x_2 - x_3 &= 3 \\ 2x_1 + x_2 + x_3 &= 12 \end{aligned}$ $(2, 3, 5)$

4. $\begin{aligned} 3x - 5y + z &= 2 \\ 2x - 3y &= 1 \\ 7x + 2y - z &= 6 \end{aligned}$ $\left(\frac{7}{8}, \frac{2}{8}, \frac{5}{8}\right)$

5. $\begin{aligned} 2x + y + z &= 1 \\ x + 2y - z &= 2 \\ x + 7y - 4z &= 5 \end{aligned}$ $(1, 0, -1)$

6. $\begin{aligned} x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 2x_2 - x_3 &= -4 \\ 4x_1 + 4x_2 + x_3 &= -2 \end{aligned}$ $(1, -2, 2)$

7. $\begin{aligned} x_1 + 2x_2 - x_3 - 2x_4 &= -2 \\ 2x_1 + x_2 + x_3 + x_4 &= 8 \\ x_1 - x_2 - x_3 + x_4 &= 1 \\ x_1 + 2x_2 + 2x_3 - x_4 &= 4 \end{aligned}$ $(1, 2, 1, 3)$

8. $\begin{aligned} x + 2y - z + u &= 9 \\ x - y + 2z - u &= 2 \\ -x + y + z - 2u &= -5 \\ 2x - y - z + u &= 2 \end{aligned}$ $(2, 3, 4, 5)$

IV. Určete k tak, aby daná soustava lineárních rovnic měla řešení:

1. $\begin{aligned} 2x_1 - x_2 + x_3 + x_4 &= 1 \\ x_1 + 2x_2 - x_3 + 4x_4 &= 2 \\ x_1 + 7x_2 - 4x_3 + 11x_4 &= k \end{aligned}$ $k = 5$

2. $\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 - 3x_2 + 4x_3 &= 4 \\ x_1 - x_3 &= k \end{aligned}$ $k = 2$

3. $\begin{aligned} x_1 + 2x_2 + 3x_3 - x_4 &= 0 \\ x_1 + 5x_2 + 5x_3 - 4x_4 &= 4 \\ x_1 - x_2 + x_3 + 2x_4 &= k \\ x_1 + 8x_2 + 7x_3 - 7x_4 &= 8 \end{aligned}$ $k = -4$

4. $\begin{aligned} 2x_1 - x_2 - x_3 + x_4 &= 0 \\ -x_1 + x_2 + x_3 - 2x_4 &= k \\ 3x_1 - x_2 - x_3 &= -1 \end{aligned}$ $k = -1$